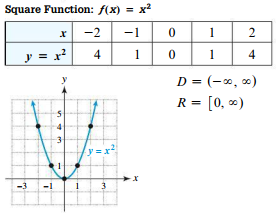
# **Lesson Objectives**

1. Parent Functions
2. Vertical and Horizontal Translations (shifts)

# **Parent Functions**

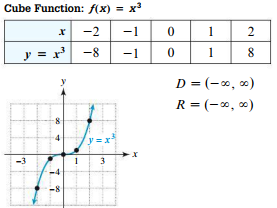
1. **Quadratic (or Square) Function:**

or



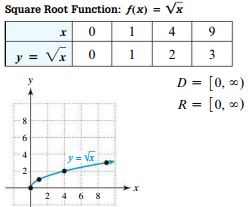
1. **Cubic Function:**

or



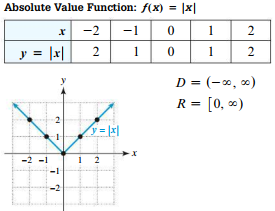
1. **Square Root Function:**

or



1. **Absolute Value Function:**

or



# **Vertical and Horizontal Shifts**

|  |  |  |
| --- | --- | --- |
| Let *f* be a function, and let *c* be a positive number. | | |
| **To Graph** | **Shift the Graph of by *c* Units** | (NOTES below) |
|  | **upward** | Adding or subtracting a number OUTSIDE parentheses with *y* causes a **VERTICAL** SHIFT in the SAME direction as that number.  **OUTSIDE – *y* is “do what you see.”** (↓,↑) |
|  | **downward** |
|  | **right** | Adding or subtracting a number INSIDE parentheses with ***x*** causes a **HORIZONTAL** SHIFT in the **OPPOSITE** direction of that number.  **INSIDE – *x* goes OPPOSITE**! (← →) |
|  | **left** |

* **EXAMPLE:** The graph of is shown with dashed (red) lines. Graph .

Choose the correct graph in solid (blue). [3.5.31]

**OUTSIDE – *y* is “do what you see.”** + 5 means **UP 5** Graph **C.**

|  |  |  |  |
| --- | --- | --- | --- |
| **A.** | **B.** | **C.** | **D.** |
|  |  |  |  |
| RIGHT 5, or *f*(*x* – 5)  INCORRECT | DOWN 5, or *f*(*x*) – 5  INCORRECT | **UP 5, or *f*(*x*) + 5**  **CORRECT** | LEFT 5, or *f*(*x* + 5)  INCORRECT |

* **EXAMPLE:** Determine which graph indicates the shift in the indicated equation. [3.5-6]

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | | The graph to the left is the given graph of *y* = *f*(*x*). | | | |
|  | | To graph *y* = *f*(*x* – 3) – 5 | | | |
| **INSIDE** parentheses:  ***x* goes OPPOSITE!**  I see – 3 with *x*, so the shift is to the **RIGHT**, not the left. | | **OUTSIDE** parentheses:  ***y* is “do what you see”**  I see – 5 outside parentheses, so the shift is also **DOWN** 5. | |
| So, together, the shift for is  **RIGHT 3, DOWN 5**. Correct answer is **B**. | | | |
| **A.** | | **B.** | | **C.** | | **D.** |
|  | |  | |  | |  |
| Left 5, Down 3 | | **Right 3, Down 5** | | Right 5, Down 3 | | Left 3, Down 5 |
| INCORRECT | | **CORRECT** | | INCORRECT | | INCORRECT |

(go on to the next page)

* **EXAMPLE:** The graph is a translation of one of the basic functions

. Find the equation that defines the function. [3.5.1]

(Type an expression using *x* as the variable. Do not simplify).

|  |  |
| --- | --- |
|  | The graph to the left is a translation of .  The INFLECTION point for is normally at the origin.  In this graph, though, it has moved: **RIGHT 4, UP 1**  RIGHT 4 is a change in ***x***, so be sure to **SWITCH** the value. (remember: **INSIDE – *x* goes OPPOSITE**!)  So, RIGHT 4 is written as **.**  UP 1 is a change in ***y***, so that is written as **+ 1**.  (remember: **OUTSIDE – *y* is “do what you see.”**)  When that goes RIGHT 4, UP 1,  the equation is *y* = |

* **EXAMPLE:** The graph is a translation of one of the basic functions

. Find the equation that defines the function. [3.5.5]

(Type an expression using *x* as the variable. Do not simplify).

|  |  |
| --- | --- |
|  | The graph to the left is a translation of .  The starting point for is normally at the origin.  In this graph, though, it has moved: **LEFT 1, DOWN 5**.  LEFT 1 is a change in ***x***, so be sure to SWITCH the value.  (remember: **INSIDE – *x* goes OPPOSITE**!)  So, LEFT 1 is written as .  DOWN 5 is a change in *y*, so that is written as **– 5**.  (remember: **OUTSIDE – *y* is “do what you see.”**)  When goes LEFT 1, DOWN 5,  The equation is *y* = . |

* **EXAMPLE:** The graph is a translation of one of the basic functions

. Find the equation that defines the function. [3.5.7]

(Type an expression using *x* as the variable. Do not simplify).

|  |  |
| --- | --- |
|  | The graph to the left is a translation of .  The vertex for is normally at the origin.  In this graph, though, it has moved: **RIGHT 2, UP 5**.  RIGHT 2 is a change in ***x***, so be sure to SWITCH the value.  (remember: **INSIDE – *x* goes OPPOSITE**!)  So, RIGHT 2 is written as **.**  UP 5 is a change in ***y***, so that would be written as **+ 5**.  (remember: **OUTSIDE – *y* is “do what you see.”**)  When that goes RIGHT 2, UP 5,  the equation is *y* = **.** |

* **EXAMPLE:** Find the equation that shifts the graph of *f* by the indicated amounts.

right 8 units, up 7 units [3.5-1]

Right 8 units is a change in ***x***, so be sure to SWITCH the value.

So, RIGHT 8 is written as . (remember: **INSIDE – *x* goes OPPOSITE**!)

Up 7 units is a change in ***y***, so UP 7 is written with **+ 7** at the end.

(remember: **OUTSIDE – *y* is “do what you see.”**)

|  |  |  |
| --- | --- | --- |
| A. |  | INCORRECT. The negative sign in front of parentheses inverts the graph upside-down, which is a reflection over the *x*-axis. |
| B. |  | INCORRECT. Similar to answer A., and + 56 also incorrect. |
| **C.** |  | **CORRECT.** |
| D. |  | INCORRECT. This graph goes LEFT 8, DOWN 7 |

* **EXAMPLE:** Use transformations to explain how the graph of *f* can be found using the graph of .

[3.5.53]

I see **– 3** with *x*, so the shift is to the **RIGHT**, not the left.

(remember: **INSIDE – *x* goes OPPOSITE**!)

So means it moves **RIGHT 3**.

I see **+ 2** outside parentheses, so the shift is also **UP 2**.

(remember: **OUTSIDE – *y* is “do what you see.”**)

So, together, the shift for from is **RIGHT 3, UP 2**.

* **EXAMPLE:** Use transformations of the graphs of or to sketch a graph of *f* by hand.

[3.5-11]

I see **– 5** with ***x***, so the shift is to the **RIGHT**, not the **left**.

(remember: **INSIDE – *x* goes OPPOSITE**!)

So means it moves **RIGHT 5**.

I see **– 2** outside parentheses, so the shift is also **DOWN 2**.

(remember: **OUTSIDE – *y* is “do what you see.”**)

So, together, the shift for from is **RIGHT 5, DOWN 2**.

|  |  |  |  |
| --- | --- | --- | --- |
| **A.** | **B.** | **C.** | **D.** |
|  |  |  |  |
| Right 5, Up 2, Inverted | Left 5, Down 2 | **Right 5, Down 2** | Right 5, Up 2, Inverted |
| INCORRECT | INCORRECT | **CORRECT** | INCORRECT |

* **EXAMPLE:** Find the equation that shifts the graph of *f* by the desired amounts.

Graph *f* and the shifted graph in the same *xy*-plane. [3.5.15]

right 5 units, upward 3 units

Right 5 units is a change in ***x***, so be sure to switch the value.

RIGHT 5 is written as: . (remember: **INSIDE – *x* goes OPPOSITE**!)

Use everywhere you see an ***x*** in the function.

changes to

Upward 3 units is a change in ***y***, so jut include a **+ 3** at the end.

(remember: **OUTSIDE – *y* is “do what you see.”**)

Combine like terms on the end

**right 5 right 5 up 3** **2 + 3 = 5**

Updated:

(Make sure BOTH sets of parentheses have the SAME value!)

Notice that in all four graphs, one of the graphs is always in the same place, with vertex at about (1,1). Remember that the overall shift is right 5 units and upward 3 units, or more simply: **RIGHT and UP**.

|  |  |  |  |
| --- | --- | --- | --- |
| **A.** | **B.** | **C.** | **D.** |
|  |  |  |  |
| LEFT and UP | LEFT and DOWN | RIGHT and DOWN | **RIGHT and UP** |
| INCORRECT | INCORRECT | INCORRECT | **CORRECT** |

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* **EXAMPLE:** Find the equation that shifts the graph of *f* by the desired amounts.

Graph *f* and the shifted graph in the same *xy*-plane. [3.5.15]

right 5 units, upward 3 units

NOTE: You can also verify the graph on your graphing calculator:

|  |  |
| --- | --- |
| Original function | Modified function: right 5, upward 3 |
|  |  |
|  |  |
|  |  |
| The graph on the calculator matches the answer we got on the previous page: |  |

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website <https://archive.codeplex.com/?p=wabbit>